Analysis of Electron-He^3 Scattering*

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Measurements of elastic electron-He^3 scattering have shown that the magnetic form factor F_M decreases less rapidly with q^2 than the electric form factor F_E in the range $0 \le q^2 \le 5$ F^{-2}. We interpret the difference as due to the effect of the exchange magnetic moment of He^3, which should have a spatial extension appreciably smaller than that of the probability density. Assuming that the static exchange moment is -0.27 nm, we determine the shape of the form factor F_x of this moment from $F_M(q^2)$ and $F_E(q^2)$. $F_x(q^2)$ can be fitted with the expression $1-(0.1\pm0.02)q^2$. Our model gives explicit predictions for the form factors in electron-H^3 scattering.

INTRODUCTION

RECENTLY, Collard and Hofstadter[1] have completed measurements of elastic scattering of electrons by He^3. Since the spin of the He^3 nucleus is $\frac{1}{2}$, they can use the standard arguments[2] to show that in Born approximation the nuclear properties can be represented by two form factors. We use the electric and magnetic form factors, F_E and F_M, each normalized to unity for $q^2=0$. We have reanalyzed the CH data using the statistical methods of our recent analysis[3] of electron-proton scattering.

In the table we show our values of F_E, F_M, their errors, and the correlation coefficient $r=\langle \Delta F_E \Delta F_M\rangle / (\Delta F_E)(\Delta F_M)$. We also show our χ^2 values for the fits by the Rosenbluth formula.

Some q^2 give very good χ^2 values; some χ^2 values are quite high. The cumulative χ^2 is 46 for 23 degrees of freedom. We do not see clear evidence for failure of the Rosenbluth formula.

In Fig. 1 we compare the CH analysis and our analysis of $F_E(q^2)$ and $F_M(q^2)$. There is agreement of the two analyses within quoted errors. Our results for the correlated error are new.

EXCHANGE MAGNETIC MOMENT

Figure 1 shows that F_M decreases less rapidly with increasing q^2 than does F_E. Collard and Hofstadter use the slope of the form factor at $q^2=0$ to give an rms radius 2.05 F for the electric charge distribution and only 1.6 F for the magnetic moment distribution. This experimental result seems paradoxical, since in each case one expected to be measuring the rms radius of He^3.

We can follow Cutkosky[4] in assuming that the measured form factor F is a product: $F=F_BF_N$. F_N is for free nucleons. F_B is for the nucleus: Nonrelativistically it is the Fourier transform of the nuclear $|\psi|^2$. We must expand our notation to include electric and magnetic form factors; we use F_N as the right linear combination of proton (p) and neutron (n) form factors for the nucleus under consideration. That is, for He^3

$$F_E=F_BF_{EN}, \tag{1}$$

$$F_M=F_BF_{MN}, \tag{2}$$

$$F_{EN}=\tfrac{1}{2}(2F_{Ep}+F_{En})\approx F_{Ep}, \tag{3}$$

$$F_{MN}=F_{Mn}\approx F_{Ep}. \tag{4}$$

Since F_{EN} and F_{MN} are approximately equal, this analysis strengthens the paradox of the marked experimental difference between F_E and F_M for He^3.

This approximation consists in limiting ourselves to the diagram shown in Fig. 2(a). The upper and lower vertices, where 2 nucleon lines join the third, combine to give the bare form factor F_B. The vertex where a nucleon interacts with the virtual photon gives the appropriate form factor F_N for free nucleons.

We wish to explain the paradoxical difference between F_E and F_M as due to a nonadditive term[4] to F_M. Such

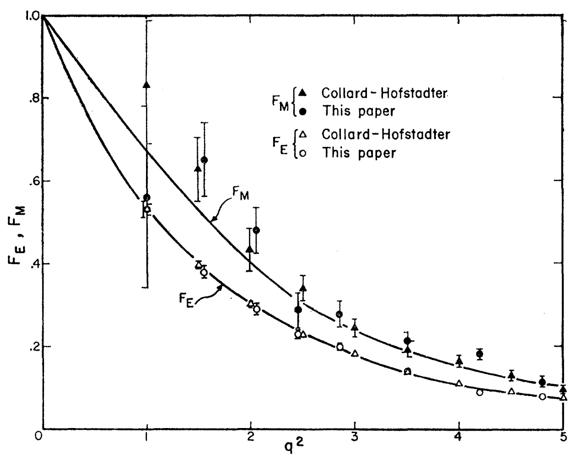

FIG. 1. Electric and magnetic form factors, F_E and F_M, plotted against four-momentum transfer, q^2, in F^{-2}. By definition, the form factors equal unity for $q^2=0$. The open triangles and circles are the Collard-Hofstadter analysis (Ref. 1) and our analysis (Table I), respectively, for F_E. The solid triangles and circles are for F_M. Empirical curves are drawn to connect the points.

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[1] H. Collard and R. Hofstadter, Phys. Rev. **131**, 416 (1963); and Bull. Am. Phys. Soc. **7**, 489 (1962).
[2] For example, S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, Oxford, 1961).
[3] M. W. Kirson and J. S. Levinger, Phys. Rev. **130**, 1549 (1963).
[4] R. E. Cutkosky, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 236.

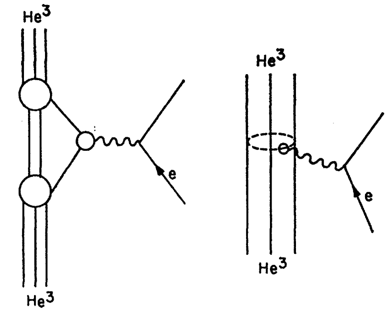

FIG. 2. Diagram (a) on the left shows the additive diagram for e-He³ elastic scattering. Diagram (b) shows the nonadditive diagram associated with the exchange magnetic moment.

a nonadditive term for the static magnetic moment of He^3 has been discussed frequently. For instance, Sachs[5] estimates that the nonadditive "exchange magnetic moment" contributes -0.27 nuclear magnetons to the static magnetic moment of He^3.

In a first approximation, the static exchange magnetic moment would be found as the difference of the observed -2.13 nuclear magnetons, and the value of -1.91 nuclear magnetons of the neutron. (That is, the He^3 magnetic moment falls outside the Schmidt line by -0.22 nuclear magnetons. The H^3 magnetic moment is 0.19 nuclear magnetons, outside the corresponding Schmidt line. The exchange magnetic moment is expected to be equal and opposite for these two mirror nuclei.) Sachs estimates the effect on the magnetic moment of mixtures of other states than ${}^2S_{1/2}$ in the He^3 ground-state wave function, and finds that a reasonable mixture increases the absolute value of the exchange moment from 0.22 to 0.27 nuclear magneton.

The exchange magnetic moment is expected[5] to be more concentrated in space than is the probability density for the ground state of He^3; and thus is in the right direction to explain the observed difference between electric and magnetic form factors. (Sachs relates the exchange moment to meson exchange between pairs of nucleons. A typical extent for the exchange magnetic moment would be the range of the two-body nucleon force, which is smaller than the rms radius of the He^3 nucleus.)

This particular nonadditive term is associated with the magnetic, rather than the electric form factor, following Siegert's theorem. The electric interaction of He^3 with a (real or virtual) photon is proportional to the electrical charge density which is, to a good approximation, proportional to the probability density of nucleons. On the other hand, the magnetic interaction includes the effects of current loops—in this case a meson current circulating among the three nucleons. Figure 2(b) shows the magnetic interaction of this circulating meson current with a virtual photon. The meson current of 2(b) may consist of pions, or of pion-resonances, such as the ρ. Any diagram other than that of 2(a) represents a nonadditive term. We postulate that the particular nonadditive term of diagram 2(b) dominates the nonadditive diagrams. The term F_B does not enter in diagram 2(b).

We wish to modify Eq. (2) by including this nonadditive term. It is more convenient to work with the nonnormalized form factors G_{MN} and G_x. The latter is the exchange moment, which we assume to have a static value of -0.27 nuclear magnetons. Then G_{MN} has a static value of -1.86 nuclear magnetons, to give the observed He^3 static moment. That is, the magnetic moment of He^3 is given by

$$G_M = F_B G_{MN} + G_x, \tag{5}$$

$$F_B G_{MN} = -1.86 F_B F_{MN} \approx -1.86 F_E. \tag{6}$$

The last equation is based on Eqs. (1), (3), and (4). Since we are interested in the shape of the exchange form factor (its static value having already been assumed), we define a normalized exchange moment form factor:

$$F_x(q^2) = -G_x(q^2)/0.27 = -(G_M + 1.86 F_E)/0.27. \tag{7}$$

We use Eq. (7) to find the shape of the exchange form factor, using Table I for F_E and F_M ($G_M = -2.13\, F_M$). The values found for F_x are given in the table. We also evaluate the error in F_x due to errors in F_M and F_E. In this evaluation it is important to include the correlated error of F_M and F_E.

We see in Fig. 3 that F_x does have a reasonably small slope as anticipated. The errors of F_x are large, especially for small q^2, due to the large errors in G_M. If, as a first approximation, we assume that $F_x = 1 - aq^2$, and make a least-squares fit we find that the slope a has the

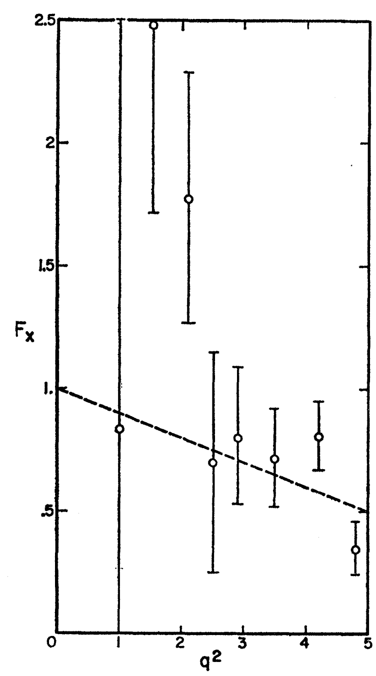

FIG. 3. The form factor $F_x(q^2)$, from Eq. (7) and Table I, plotted against squared four-momentum transfer in F^{-2}. The least-squares line for a linear dependence is shown.

[5] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts 1953).

TABLE I. Analysis of electron-He^3 scattering.[a]

q^2 in F^{-2}	F_E	Error	F_M	Error	$r(\%)$	χ^2	Degrees of freedom	F_x	Error	F_B
1.0	0.530	0.019	0.56	0.22	−73	10.0	4	0.84	1.8	0.60
1.55	0.380	0.016	0.65	0.089	−71	1.4	3	2.49	0.77	0.45
2.05	0.290	0.013	0.480	0.057	−75	2.2	4	1.78	0.51	0.37
2.45	0.230	0.010	0.291	0.052	−71	1.0	2	0.70	0.45	0.31
2.85	0.199	0.0085	0.277	0.031	−72	3.6	3	0.81	0.28	0.27
3.50	0.141	0.0070	0.215	0.021	−75	6.4	3	0.72	0.20	0.20
4.20	0.090	0.0070	0.182	0.014	−70	13.3	2	0.81	0.14	0.14
4.80	0.0805	0.0039	0.115	0.012	−70	8.5	2	0.35	0.11	0.13
					Total	46.4	23			

[a] Columns 2 to 6 give the electric and magnetic form factors, F_E and F_M, their statistical errors, and r, the coefficient of correlation of their errors The comparison between χ^2 values and the degrees of freedom checks the validity of the Rosenbluth formula. The exchange magnetic moment F_x is found from Eq. (7), while the He^3 "bare form factor" F_B is found from Eq. (8).

value of $(-0.10\pm0.02)F^2$. The χ^2 value for this fit is 12 for 7 degrees of freedom—this large a χ^2 value has a probability of 10% so the assumption is not inconsistent with present data.

From Eqs. (1) and (3) we obtain the approximate result for the bare form factor,

$$F_B=F_E/F_{Ep}. \quad (8)$$

The values for this form factor are also given in the table. Of course, this treatment is identical to that of CH for evaluation of the rms radius of He^3, where they correct for nucleon size. However, our method also allows a very simple, approximate correction for nucleon size throughout the entire range of q^2. (We note, without explanation, that all 8 values of F_B for $1\leq q^2\leq 4.8F^{-2}$ agree, within 13%, with the simple formula $F_B=0.68/q^2$. Obviously this formula does not hold for the static case.)

DISCUSSION

The above analysis seems to us quite plausible, but is not conclusive evidence that the exchange magnetic moment does have a form factor F_x similar to that anticipated by Sachs and others. Further evidence to support our interpretation could come from two different approaches. First, it might be possible to develop a quantitative theory of the exchange moment, so that we would have a prediction of $G_x(q^2)$ to compare with the above result based on our analysis of e-He^3 scattering. Second, our analysis predicts results for e-H^3 elastic scattering, which can be compared with future experiments.[6]

The term F_B should be the same as for He^3. (Coulomb effects in He^3 could be treated as a small perturbation, if necessary.) The value of F_{EN} may be somewhat different for H^3 than for He^3. We should replace Eq. (3) for He^3 by

$$F_{EN}(H^3)=F_{Ep}+2F_{En}. \quad (9)$$

The ratio of the H^3 and He^3 electric form factors would provide another method of finding the elusive charge form factor of the neutron since

$$F_E(H^3)/F_E(He^3)=(F_{Ep}+2F_{En})/(F_{Ep}+\tfrac{1}{2}F_{En}). \quad (10)$$

From the mirror theorem,[4] the value of $G_x(q^2)$ should be the same for H^3 as for He^3, except for sign. This gives the prediction

$$G_M(H^3)=2.71F_E(H^3)+0.27F_x. \quad (11)$$

(The coefficient 2.71 is chosen to give the observed H^3 static moment of 2.98 nm.) The value of F_E for H^3 comes directly from the e-H^3 scattering; the value of F_x comes from the table.[7]

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[6] See Collard *et al.*, Phys. Rev. Letters **11**, 132 (1963).

[7] We could instead use the mirror theorem alone to assert that the average moment form factor $\bar{F}_M$ should have very nearly the same slope as $F_E(He^3)$ or $F_E(H^3)$. Here $\bar{F}_M=[G_M(He^3)+G_M(H^3)]/0.85$.